Exercise 72

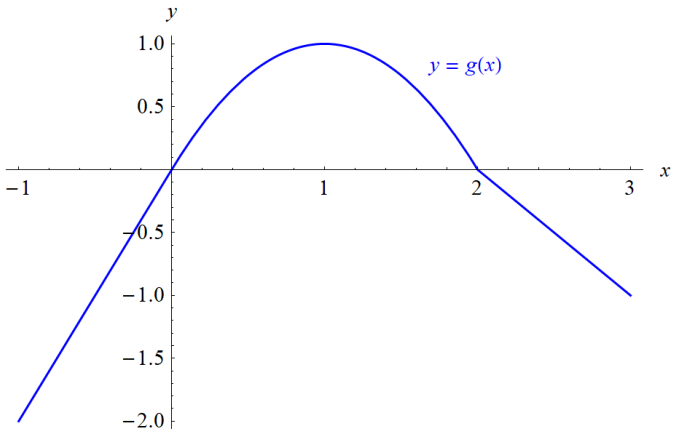
At what numbers is the following function g differentiable?

$$g(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ 2x - x^2 & \text{if } 0 < x < 2 \\ 2 - x & \text{if } x \geq 2 \end{cases}$$

Give a formula for g' and sketch the graphs of g and g'.

Solution

Below is a graph of $g(x)$ versus x.



Although the function is continuous, there's a kink in the curve at $x = 2$, which means its slope (or derivative) is undefined there. That is, g is not differentiable at 2. The derivative of g is

$$g'(x) = \begin{cases} 2 & \text{if } x \leq 0 \\ 2 - 2x & \text{if } 0 < x < 2 \\ -1 & \text{if } x > 2 \end{cases},$$

and its graph versus x is shown below.